Speeding Up The Estimation Of Floated Ambiguities For Sub-Decimeter Kinematic Positioning At Sea

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BIOGRAPHIES

Dr. Oscar L. Colombo works on applications of space geodesy, and has developed and tested methods for highly accurate, very long baseline, kinematic GPS positioning. He has a degree in Telecommunications Engineering from the National University of La Plata, Argentina, and a Ph.D in Electrical Engineering from the University of New South Wales, Australia.

Dr. Alan G. Evans has been working in Global Positioning System (GPS) applications at the Naval Surface Warfare Center, Dahlgren Division (NSWCDD) since 1981. He received the B.S.E.E. degree from Widener University in 1964, and the M.S. and Ph.D. degrees from Drexel University in 1967 and 1971, respectively.

Prof. Masataka Ando is a seismologist who got his first degree from Tokyo University in 1967 and his Ph.D. in 1974. He started seismological research at the Disaster Prevention Research Institute of Kyoto University. Since 2000, he has been Professor of Seismology at the Center for Seismology and Volcanology of Nagoya University, where he studies the mechanisms of large earthquakes, and the structure of the crust and upper mantle. Recently, with his group at Nagoya University, he has been developing an ocean-bottom positioning system for monitoring crustal deformations associated with subduction earthquakes.

Dr. Keichi Tadokoro is a seismologist who got his first degree from Kobe University and his doctorate from Kyoto University in 2000. He is an Associate Professor at the Center for Seismology and Volcanoloy of Nagoya University. His work includes developing an ocean-bottom system for monitoring the crustal deformation associated with subduction earthquakes, and the study of healing, recovery, and strain-accumulation processes in fault zones.

Mr. Kazutoshi Sato got his first degree from Chiba University, in 1999, and started post-graduate studies in seismology at Kyoto University in 2000. He is now an exchange student at Nagoya University.

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ABSTRACT

Using carrier phase DGPS to find the precise position of a ship or a buoy in the high seas presents the problem that phase ambiguities cannot be resolved with confidence over very long baselines. The ambiguities may be "floated" instead. However, the convergence of the Kalman filter in a kinematic solution with floated ambiguities (i. e. solving for real-valued Lc biases) is often quite slow. But it could be speeded up by imposing the dynamic constraint that on a ship, or a buoy, changes in elevation are likely to be quite gradual, once waves and faster variations are smoothed out. To test this idea, two GPS data sets, one from a buoy in the US and another from a ship in Japan, have been processed in differential kinematic mode, floating the ambiguities with and without a constraint on the mean height. Each set contains several hours of uninterrupted data from a rover and from a nearby coastal site. In the US, the site was in Duck, North Carolina; in Japan, it was in Yaizu. In the US, high-rate data from distant receivers (350 km-1100 km) were also available for long-range solutions.

INTRODUCTION

Getting precise kinematic results quickly, in a few epochs, is crucial in real-time applications, and in the post-processing of data sets that are short, or have frequent breaks with long gaps. Fast convergence is usually obtained by fixing the phase ambiguities whenever the vehicle is within 10-20 km of a base station. But with a ship, buoy, etc. in the high seas, achieving sub-decimeter precision requires "floating the ambiguities", a much slower process. A possible remedy investigated here exploits the fact that a ship is on the surface of the sea. Finding a practical solution has important consequences:

Precise positioning with GPS of buoys in the deep ocean can be of help to those studying tides, waves and currents, charting the sea-floor and the marine environment with advanced forms of remote sensing, or calibrating satellite-born altimeters, to map with them the sea surface. In real-time, it may help detect tsunamis while still very far from the coast, giving earlier warning to those at risk [1].

In ships, a fast and precise GPS technique may enable safer and more efficient marine navigation. It should also enhance ship-borne remote sensing for high-resolution ocean floor mapping, among other applications.

As explained later in this paper, one potentially valuable application of ship positioning is in the study of earth-crust deformations and tectonic movements in the sea floor, to understand the forces that build continents and cause volcanoes and earthquakes.

KINEMATIC DATA ANALYSIS

The results shown in this paper were obtained by sequential processing of the carrier phase with a Kalman filter, with and without optimal smoothing. Short-baseline solutions were used for comparison purposes, and were obtained resolving the L1 and L2 phase ambiguities. For the long-baseline solutions, the observations were double-differenced between the rover and each reference receiver, and combined to form the ionosphere-free observable Lc. The Lc biases (a linear combination of the L1 and L2 integer ambiguities) were estimated as real numbers (i.e. "floated"), along with other nuisance unknowns. All this is standard procedure in long-baseline GPS solutions. (Recently, there have been successful attempts at resolving the L1 and L2 ambiguities with the roving receiver hundreds of kilometers away from any base station, using computed tomography to model and then correct the effect of the ionosphere on GPS data [2].)

Unknowns of some or all of the following kinds were estimated simultaneously [3], [4], [5]: (a) Corrections to the vehicle *a priori* known position (treated as three "white noise" states, with a 100 m *a priori* one-sigma precision per coordinate). (b) The biases in the Lc combination (treated as constants, each with a 10m *a priori* sigma). (c) Error in calculated tropospheric correction at each site (a constant plus a slow random walk error in the zenith delay). (d) GPS satellite orbit errors, as pseudo initial state errors plus small (10^{-8} m/s^2) acceleration errors, using analytical orbit perturbation partials. The *a priori* sigma for the initial position and velocity were chosen according to the SP3 Format file headers for precise IGS SP3 orbits, or equal to 1 m and 0.1 m/s for broadcast orbits. Altogether, as many as about 100 error states were estimated in each case. (To save computations and memory, orbit and bias states no longer active, were "recycled", yielding their places to newly active ones). Compressing data into 2-minute averages shortened calculations and allowed the use of the mean height constraint. The results were obtained with GPS software developed by the first author. It runs under UNIX, LINUX, Windows 95, 98, ME, NT, and 2000. The calculations were made in the 266 MHz-Pentium II laptop used to write the final draft of this paper. For the US test, it took less than one minute to process three hours' worth of data at 0.2 Hz from three receivers.

SPEEDING UP KALMAN FILTER CONVERGENCE

The Kalman filter has to assimilate enough data to converge to a precise solution. The time needed for this should be kept as short as possible, since a tsunami (for example) could pass unnoticed while the estimated height of a buoy is still not precise enough to detect it. While clearly needed in real time, fast convergence is always desirable. Even in post-processing, frequent gaps in GPS reception may cause the filter to be re-initialized too often, preventing its proper convergence, and resulting in a filter/smoother solution that is not precise enough. (The final level of precision achieved with the filter is that of the post-processed trajectory calculated with the smoother.)

A kinematic solution wisely ignores the often poorly known dynamics of the vehicle. In the case of a craft floating on water, however, the use of a slow-varying mean height constraint can shorten the convergence transient without introducing unwarranted assumptions as to how that craft otherwise moves. For a small buoy, the running average of its height, corrected for the solid earth tide, should approximate the wave-filtered, time-varying height measured with a tide-gauge, which changes gradually and predictably with time (e.g. Figure 6). In the long-range technique used here a constraint on the mean height is easy to implement, because of the use of *data compression* (averaging) to speed up calculations and economize other computer resources, such as hard disk space for scratch files [5]. The mean position of the vehicle, averaged over several minutes, is estimated before the instantaneous position. Given this, it is easy to create pseudo-observations of the form:

Mean height (estimated) - mean height (model)
= Error in model (constant + random walk) + noise.

The "model" is the known value of the time-varying sea level at the location of the vehicle. It is the sum of the long-term mean sea level, the geocentric tide (ocean tide + solid earth tide + ocean loading) and the inverted-barometer effect of atmospheric pressure (good models of mean sea level and tides are available for most of the oceans from the analysis of satellite altimetry). The model can be improved, over time, using the GPS-determined ship or buoy heights.

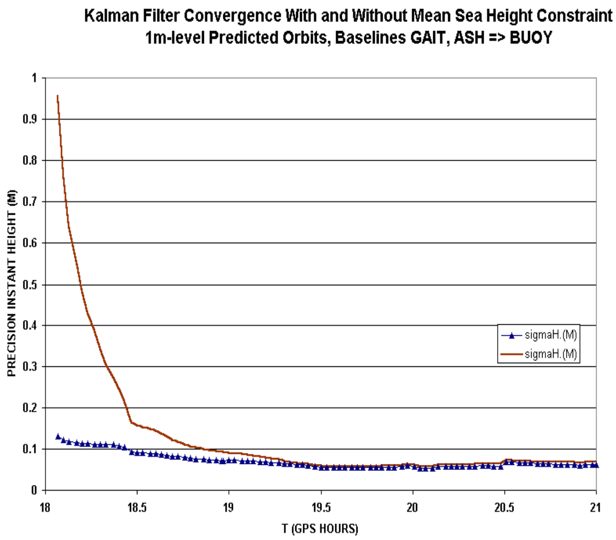

Figure 1. Convergence of the Kalman filter solution with and without the proposed mean height constraint.

For this study, the "model" height was a constant equal to the initial 6-minute average height according to the precise short-range solution. On the right-hand side of the equation, the unknown constant represents the model's initial error, and the random walk represents the model's error in height change.

The "noise" is the residual wave action left in the average. For an averaging interval T_a seconds long, given approximately sinusoidal waves of dominant period T_w with peak-to-null amplitude A_w, and a data rate high enough to keep waves from being aliased into mean sea height changes, the value of the residual wave-effect N_w is:

$$N_w \leq T_w / (2^{3/2} \pi T_a) \; A_w \quad \text{(r.m.s.)}$$

To be conservative, "≤" could be replaced with "=". Choosing: T_a = 120 seconds (a good compression interval for the solution, if not for averaging all possible waves), A_w ~ 2 m, and T_w ~ 20 seconds, then N_w ~ 4 cm (r.m.s.). Waves at the time of the test were much smaller, but this choice of amplitude was judged more realistic for open waters. The other (one sigma) uncertainties were chosen as follows: Unknown constant, 10 cm (for mean sea surface and tide models as good as those from satellite altimetry); random walk system noise, 6 cm /(min)$^{1/2}$ (a one-sigma change in mean height of almost 20 cm in 10 minutes.)

The expected convergence of the Kalman filter can be seen in Figure 1. The Figure shows the formal precision (one sigma), in meters, of the estimated *instantaneous* height as a function of time: (1) For a purely kinematic (unconstrained) solution (continuous line), and (2) for a mean-height constrained solution (triangles). Since the filter is supposed to be operating in real time, the GPS satellite orbits have been given *a priori* uncertainties of 1 m in each initial coordinate.

This assumes the availability of reasonably good predicted orbits, and that the errors in those orbits are also estimated in the filter (orbit relaxation or adjustment).

The predicted orbits may be calculated at a central site, using data from a very-large-area network of GPS stations, or else might be obtained from some future international service, perhaps based on the IGS.

As seen in Figure 1, the convergence in height should clearly improve with the constraint. As it shall be seen, the convergence in *horizontal* precision also improves markedly. This happens because the height constraint increases the precision of the estimated Lc biases, and so it helps to determine better the distance to the satellites. This, in turn, improves precision in all directions, including the horizontal.

TEST OFF DUCK, NORTH CAROLINA, USA

Test Setup. The buoy test took place on 26 October 1999, at the initiative and under the direction of Dr. Alan G. Evans, of the Naval Surface Warfare Center, Dahlgren Division (NSWCDD), at the Army Corps of Engineers Field Research Facility (FRF) in Duck, North Carolina. GPS dual-frequency receiver data were collected at a buoy (site "BUOY") anchored at the seaward end of the very long FRF pier, and at a reference site atop a building ("FRFR"), 500 m away, near the pier's landing.

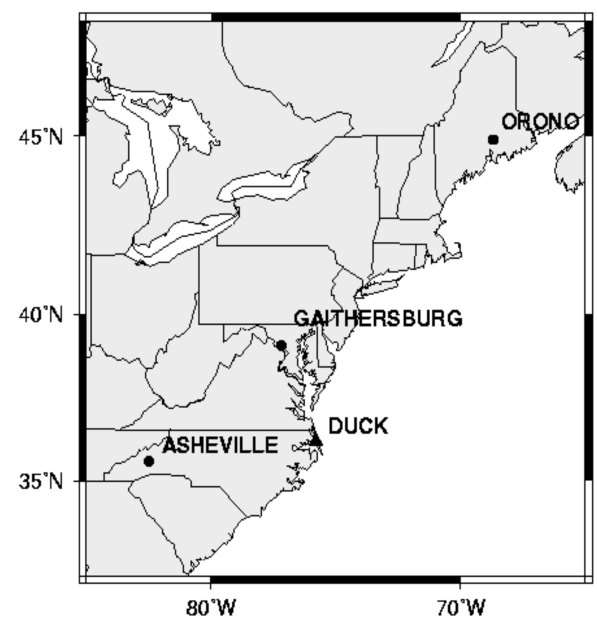

Figure 2. Duck and the distant CORS GPS sites. Duck is 352 km from Gaithersburg, 617 km from Asheville, and 1138 km from Orono.

Figure 3. The Duck FRF pier seen from the top of a nearby tower.

Since the nature of what lies between receivers hardly affects GPS results, instead of using distant reference sites across the sea, it is just as valid to test the idea of positioning buoys in the deep ocean using readily available reference receivers installed far *inland.* So reference receiver data were obtained from some of the Continuously Operating Reference Stations (CORS) run by the National Oceanic and Atmospheric Administration (NOAA) in the USA. CORS data are publicly available on the Internet as RINEX Format files, stored at five-second intervals. The data were downloaded from stations in Gaithersburg ("GAIT"), Maryland, in Asheville ("ASHE"), North Carolina, and in Orono ("ORO1"), in Maine. These were situated 352 km, 617 km, and 1138 km away from Duck, respectively (Figure 2). Aspects of the test setup at Duck are shown in Figures 3, 4, and 5.

The "FRFR" reference site in Duck was put in the reference frame of CORS by making a precise static solution for it with the coordinates of the CORS sites fixed to their published values corrected for tectonic motion. A total of four hours of data were collected at the buoy, but only the last three hours were processed, because of initial reception problems. Results from this test have been reported in [6]. Recomputed with improved software, they complement the ship results of next section.

Figure 4. The buoy deployed near the end of the pier.

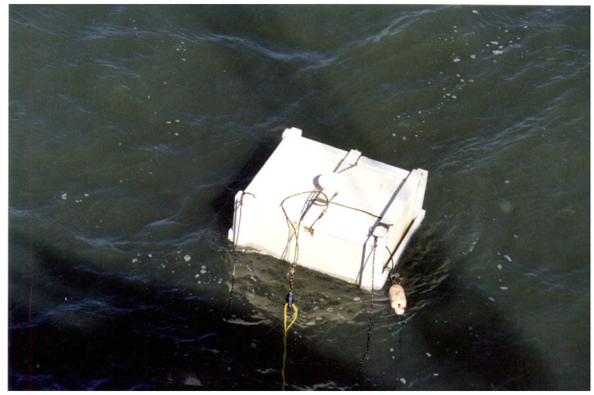

Figure 5. Close-up of the buoy showing the small, round GPS antenna on top. (Dimensions: 4' x 4' x 2')

The distant sites were used as base stations for a long-range kinematic solution. This was made at 5-second intervals, (the CORS data rate), and then compared to a short-range solution relative to "FRFR", near the pier's landing. As shown in Figure 6, a running average of the observed instantaneous buoy height, with a window of 5 or 6 minutes duration, largely eliminates the short-term fluctuations due to ordinary waves (with periods of 5 to 30 seconds). This reveals more gradual changes in water level, such as tides and deep-ocean tsunami. The instantaneous height accuracy is a few centimeters. Such accuracy is possible because the differential effect of the ionosphere on the data cancels itself out over sufficiently short baselines (less than 10 km), making it possible to resolve exactly the phase ambiguities. To get the highest accuracy in this short-range solution (500 m), only the unambiguous L1 phase was used.

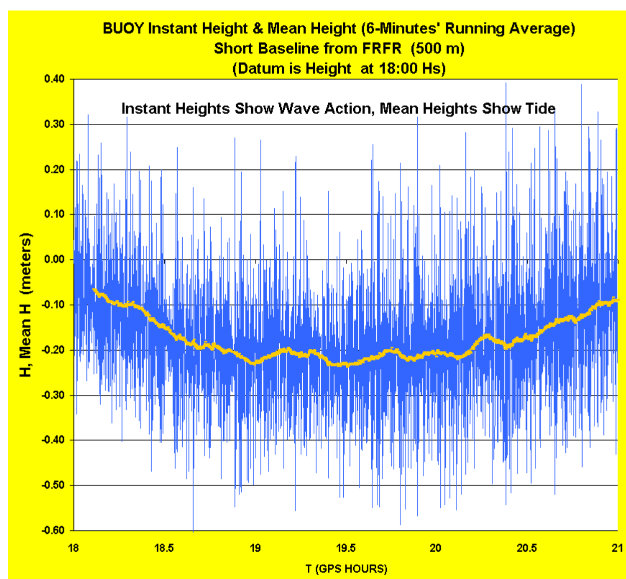

Figure 6. Waves and tide during test at Duck, North Carolina, as observed with GPS on a buoy. From a short-baseline differential solution, with L1 and L2 carrier phase ambiguities resolved (used as "truth" in the study).

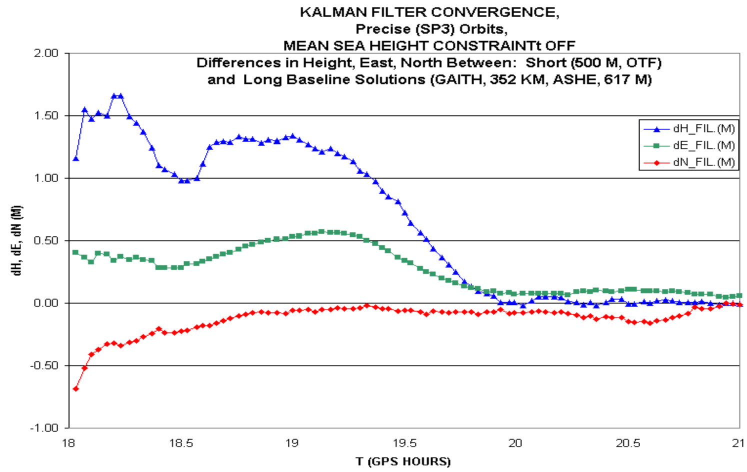

Figure 7. Difference between short-range control solution and long-range relative to ASHE and GAIT, without mean height constraint. Precise IGS orbits used.

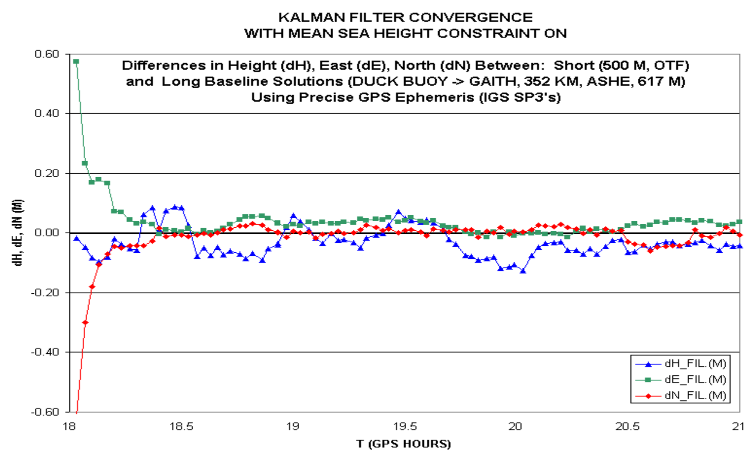

Figure 8. As in Figure 7, but using the mean height constraint. Convergence is faster for all three coordinates, not just height. Precise IGS orbits used.

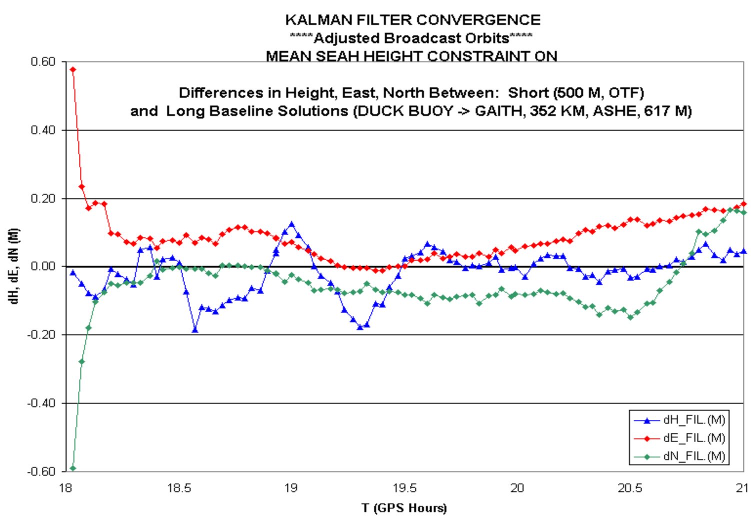

Figure 9. Using broadcast ephemeris while estimating their errors.

Duck Test Results. As explained in more detail in [6], a tide gauge was located nearby, providing a record of the change in sea level in good agreement with the running average plotted in Figure 6 above. Furthermore, the running averages of the long-range solutions relative to Gaithersburg (352 km away), Asheville (617 km), and Orono (1138 km) all agreed with the tidal record within a few centimeters. Those were filter plus smoother solutions (full Bayesian least squares processing).

Figure 7 shows the differences in height ("dH"), longitude ("dE"), and latitude ("dN") between the unconstrained Kalman filter solution relative to ASHE and GAIT, and "truth": The short-range solution relative to FRFR.

Notice the very slow convergence in this case: It takes about two hours before all three coordinates of the long-range solution are within 10 cm of the short-range results.

The effect of imposing a mean height constraint is shown in Figure 8. Notice that all three coordinates show a clearly faster convergence than without the constraint.

In Figure 9 one can see results comparable to those in Figure 8, but using the ephemeris broadcast with the GPS Navigation Message, which are less precise than the final (SP3 Format) orbits of the IGS. The broadcast orbit errors are estimated together with the buoy trajectory, the carrier phase Lc biases, and the tropospheric refraction correction errors [5]. Comparing Figures 8 and 9, one notices that the position errors for the buoy when adjusting the broadcast orbits are roughly twice the size those with the precise IGS orbits (without adjusting the broadcast orbits, the errors are considerably larger). In real-time operation, one may expect to have worse orbits than those of the IGS, but better ones than the broadcast ephemeris. Data from the receivers of a continuously operating network can be used to estimate the errors in the broadcast ephemeris, and the result can be radioed as corrections to users in the area.

To see to what extent the use of the height constraint biases the height in the long-range solution, constrained results fully processed (filtered and smoothed) were compared to their unconstrained counterparts. The three-dimensional r.m.s. and mean values of their differences, both for the Duck buoy test and for the Yaizu ship test in next section, appear in Table 1. For Duck, those numbers are: 3.8 cm (3-D r.m.s.), and 2.2 cm (3-D mean). These are comparable to the differences between each trajectory and the short-baseline control, or "truth". No significant biases appear to have been created by the use of the mean height constraint.

TEST OFF YAIZU, JAPAN.

Background. The group of Prof. Ando, at the Center for Seismology and Volcanology, Nagoya University, is developing a system combining GPS, and underwater acoustic positioning to locate precisely points on the sea-floor [7], [8].

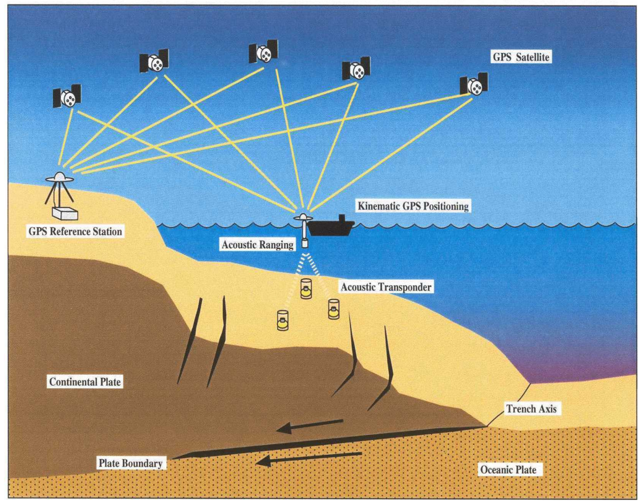

Figure 10. Using GPS and acoustic ranging to find the position of a site on the seafloor (in this case, an imaginary point inside the triangle defined by three transponders). Also shown: Ocean crust pushing in under continental crust, which develops cracks (or faults) under the stress.

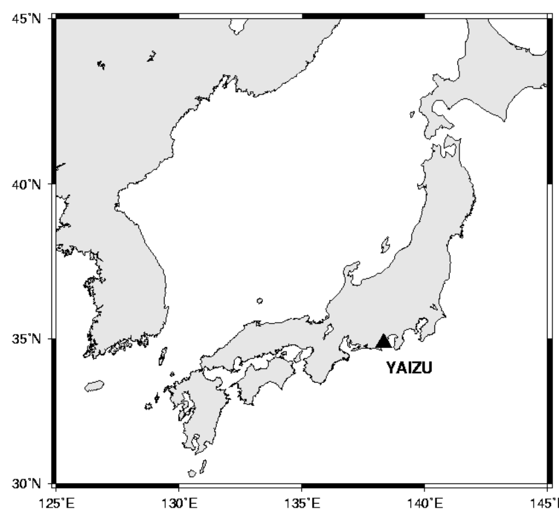

Figure 11. The reference site in Yaizu.

The objective is to conduct repeated surveys to monitor movements in the ocean crust around Japan associated with inter-seismic, seismic and post-seismic deformations. These are mainly due to stresses created by the sinking, or subduction, of the Philippine tectonic plate under the Japanese islands.

With the system under development, the positions of sea-floor stations (with acoustic transponders) are obtained from a ship, as shown schematically in Figure 10, above.

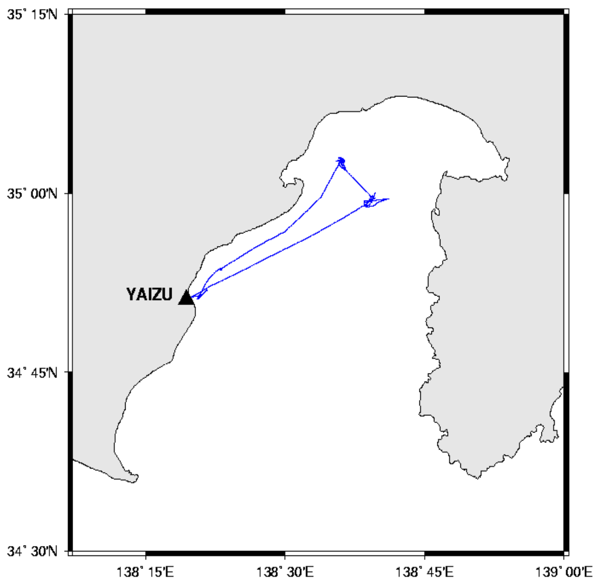

Figure 12. Ship track for 16 January 2001. (Followed counterclockwise.)

First, the position and orientation in space of the vessel are determined to high precision with help from kinematic GPS, and then the ocean-bottom stations are located by acoustic ranging from the vessel, which circles them, keeping a horizontal distance comparable to their depth (up to several kilometers). An important and difficult step is the precise correction for changes in the speed of sound along the path of the signals. This requires measuring water salinity and temperature all the way from the surface to the bottom. Another difficulty is that the ocean-bottom stations could be hundreds of kilometers from shore, and from the nearest GPS reference site. This means floating ambiguities, so here the height constraint may be useful.

Figure 13. The "Suruga Maru".

Figure 14. View of Mount Fuji from Yaizu harbor, with GPS antenna.

In a recent test, three transponder stations were dropped in free fall from the ship to the bottom. Attempts were then made at locating them by acoustic ranging. The position of one seafloor station was found with the estimated horizontal and vertical precision of 5 cm and 15 cm., respectively.

GPS Test Setup. The GPS data used in this study were obtained in the second day of an offshore experiment that took place on January 15th and 16th, 2001. The reference receiver was in the home harbor of the "Suruga Maru" (Figure 13), in the coastal city of Yaizu (Figure 14), about 30 km from the area of the test (Figures 11 and 12). The trip on the 16th lasted 22 hours, and Figure 12 shows the complete track of the ship. Figure 15 shows the ship GPS antenna installed on the prow, with a conic radome. Data were collected continuously at 2 Hz during the 22 hours. Five of those hours will be considered here. The track for that period is shown in Figure 16. During those five hours, the "Suruga" moved steadily at 11 knots for the first hour, and wandered quite slowly for most of the other five.

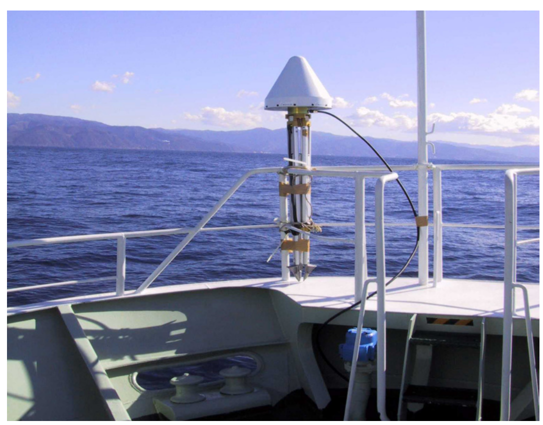

Figure 15. GPS antenna setup on the "Suruga".

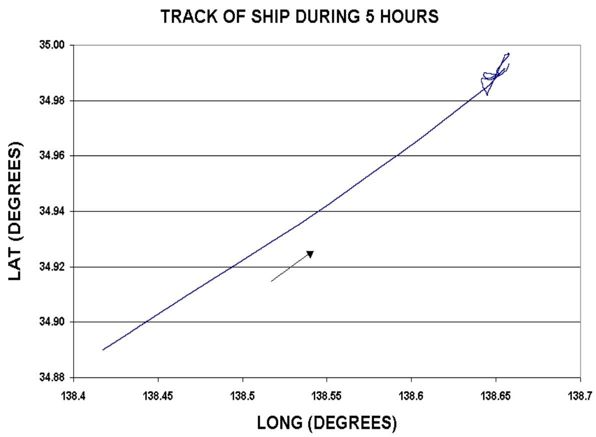

Figure 16. Ship track for the period analyzed in this study.

Effect of Ship Velocity on Antenna Height. Figure 17 shows the instantaneous height of the antenna, from kinematic GPS, showing mainly wave-like changes (with periods of less than ten seconds, greatly compressed at the scale of the plot). A 2-minute running average reveals a gradual, tide-like change, with occasional dips. It might be thought that these are caused by data glitches or by bad data processing. But closer inspection shows that they happen only while the ship moves with some speed (Figure 18), and to be proportional to that speed. They are not artifacts, but represent a true decrease in the height of the antenna, caused by the ship riding lower on the water or changing its trim as a result of hydrodynamic forces. This effect is not found in passively floating objects such as buoys. So it has to be considered when applying the mean height constraint to a ship, for example by increasing the random-walk system noise.

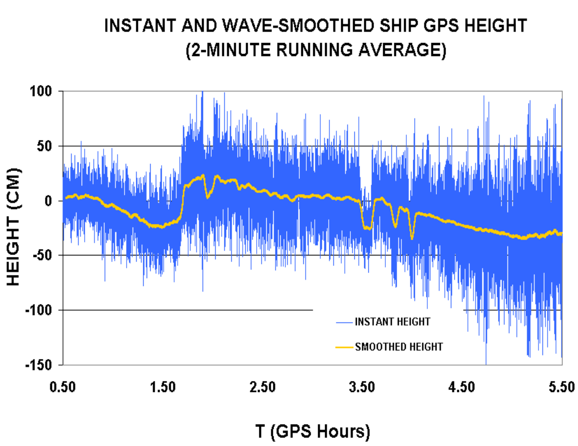

Figure 17. Instantaneous antenna height (waves). A running average mostly follows a gradual change in sea level (tide), at times dipping appreciably lower.

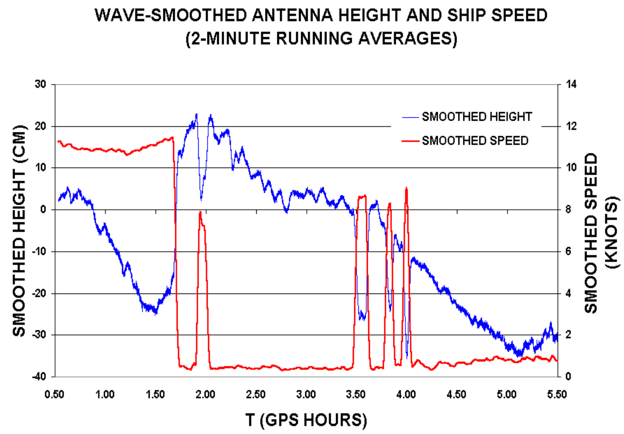

Figure 18. Running averages of height and total horizontal velocity. The ship dips when moving at significant speed. From 1.7 hours on, the ship moves slowly, mostly.

Yaizu Test Results. The five hours of GPS data (from 0.5 to 5.5 GPS hours) selected for this study were used in kinematic solutions with and without the mean sea constraint. The data were collected without interruptions, and they were processed (a) as such, (b) with a simulated data break of 20 minutes, after which the filter was restarted. These are two situations likely to occur in practice. The results are plotted in the figures that follow, and summarized below in Table 1. Since the baseline length was less than 40 km, the type of orbits used (broadcast or SP3) makes no difference, and there is no need to adjust them. So the IGS SP3 orbits for the day were used, unadjusted.

TABLE 1
FULLY PROCESSED SOLUTIONS
WITH /WITHOUT MEAN HEIGHT CONSTRAINT
COMPARED TO "TRUTH"
AND TO EACH OTHER

SOLUTIONS	3-D RMS	3-D MEAN
DUCK, UNCONSTR.	4.1 CM	6.4 CM
DUCK, CONSTRAINED	3.6 CM	5. 0 CM
YAIZU, UNCONSTR.	0.3 CM	0.0 CM
YAIZU, CONSTRAINED	0.8 CM	0.1 CM
DUCK, CONSTR.-UNC.	3.8 CM	2.2 CM
YAIZU, CONSTR.-UNC.	0.8 CM	0.1 CM

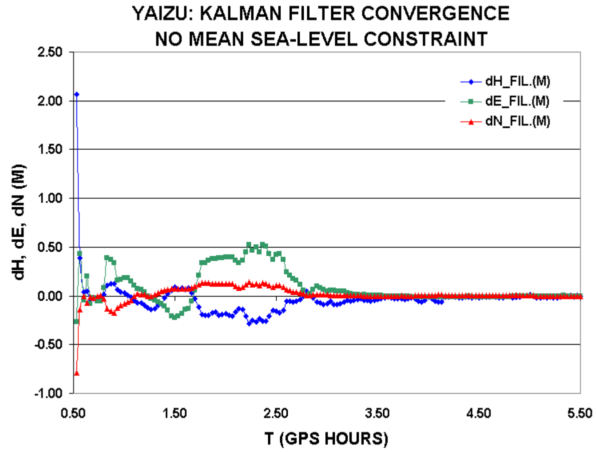

Figure 19. Unconstrained solution versus "truth".

Uninterrupted Data. Figure 19 shows the difference between the unconstrained Kalman filter solution with floated ambiguities and the control solution with ambiguities resolved. Figure 20 shows the same type of plot, but with the constraint applied. These Figures resemble those for the buoy test, except that convergence in longitude ("dE") here takes considerably longer than for height ("dH") or latitude ("dN"). A few minutes after starting the run, all three components are already considerably closer to "truth" with the constraint than without it (each point in the graph represents a 2-minute interval). Figure 21 shows the same height differences plotted in Fig. 20, making it easier to see that within 2 minutes (the first point) of starting the solution, the constrained height already is less than 10 cm, and that it stays so virtually the whole time. To see to what extent the use of the constraint might bias the results, two fully processed solutions (filter plus smoother), with and without the constraint, were compared to "truth", and with each other.

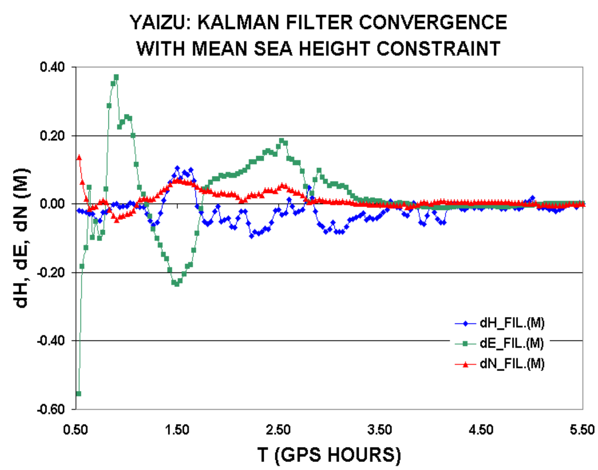

Figure 20. Constrained solution versus "truth".

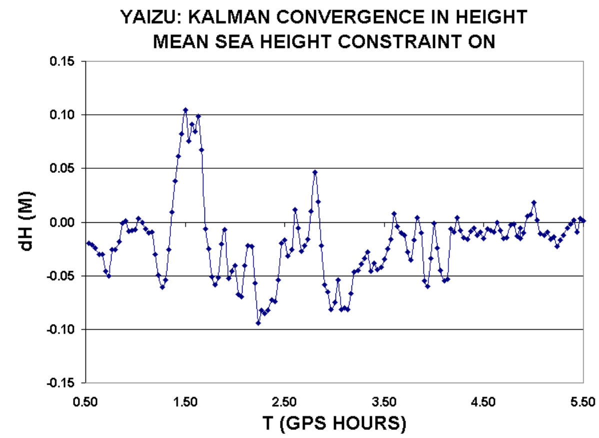

Figure 21. Filter convergence in height.

The statistics of these comparisons for the uninterrupted data case are shown in Table 1, alongside those for the buoy. The numbers for the "Suruga" are smaller, mainly because over a longer interval (5 hours vs. 3) the floated ambiguities can be solved more precisely. In particular, the mean 3-D difference with "truth" is too small to register with only one decimal place. All these numbers are well under ten centimeters, so there is no indication of significant biases, at the decimeter-level, when using the constraint.

Interrupted Data. The main conclusion for the uninterrupted case also holds true here: Convergence obviously improves with the use of the mean height constraint. The plots in Figures 22 and 23 are identical with their counterparts for the uninterrupted case, Figures 19 and 20, up to the time of the break in the data. Convergence after the break is faster, particularly in longitude. The receiver-satellite geometry was better (lower PDOP) in the second half of the 5-hour period, than in the first.

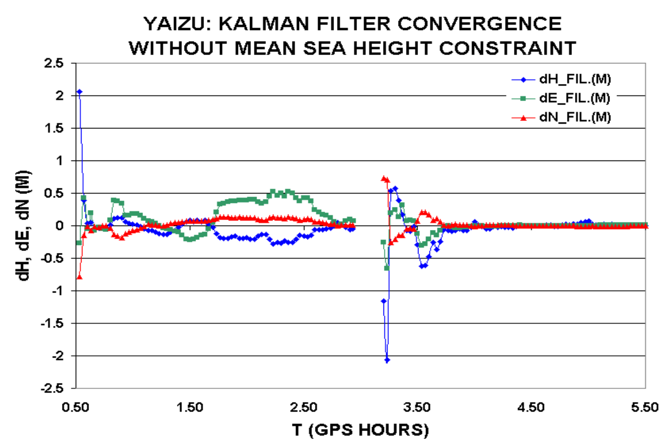

Figure 22. As in Figure 19 (no constraint), but with filter re-started after a simulated data break of 20 minutes.

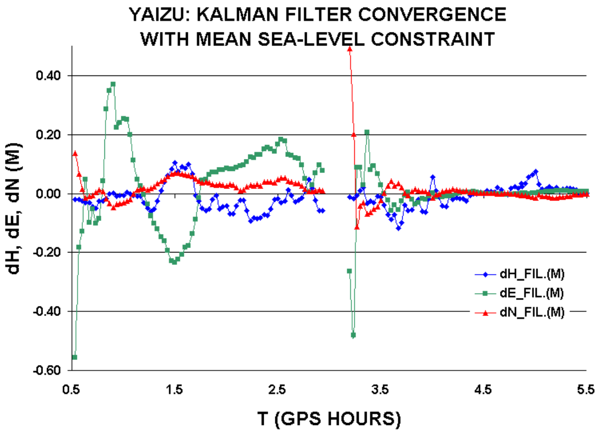

Figure 23. Same as in Fig. 22, but with constraint

CONCLUSIONS

Two separate experiments in long-range kinematic positioning at sea are reported here, one with GPS data collected in the US with a buoy, the other, in Japan with a ship. In each, the estimated trajectories have been compared to "truth": A carrier phase navigation solution relative to a nearby site and with L1 and L2 ambiguities resolved.

Based on the results, these points can be made:

(1) The use of a slow random-walk constraint on the dynamics of the mean height of the antenna, over a time-window long enough to average out waves (2-5 minutes), significantly speeds up the convergence of the navigation filter when floating carrier phase ambiguities. (i.e., estimating Lc biases.) The type of craft (buoy or ship) or its mode of operation (floating, drifting, or travelling at a steady speed) does not affect this conclusion.

(2) Agreement between the fully processed solution (i.e., with filter and smoother) and "truth" is at the level of a few centimeters in both 3-dimensional r.m.s and 3-D mean, over the whole duration of each test (3 hours and 5 hours).

(3) The use of the constraint biases the results below the decimeter level, as indicated by the discrepancies between the fully processed solutions with and without the constraint, and between each of them and "truth".

(4) In the US test, the distances to some far GPS sites ranged from 300 km to 1100 km, a situation comparable to having a ship in the high seas. In both tests, agreement after convergence between all Kalman filter solutions and "truth" is at the sub-decimeter level. Smoothed US results agree within centimeters with the change in sea level observed at a local tide gauge.

(5) In the US test, separate solutions were made using the precise IGS SP3 orbits and the broadcast GPS ephemeris.

The results show it is possible to attain a high level of precision over long baselines, even with broadcast orbits, if orbit errors are estimated as part of the navigation solution. This last conclusion is particularly relevant to real-time applications.

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REFERENCES

[1] Kato, T., Y. Terada, M. Kinoshita, H. Kakimoto, H. Issiki, Real-Time Observation of Tsunamis by RTK-GPS, *International Symposium on GPS on Applications to Earth Sciences and Interaction with other Space Geodetic Techniques*, Tsukuba, Japan, October 1999.

[2] Colombo, O.L., M. Hernández-Pajares, J.M. Juan, J. Sanz, and J. Talaya, "Resolving carrier-phase ambiguities on-the fly, at more than 100 km from nearest site, with the help of ionospheric tomography", *Proceedings ION GPS'99*, Nashville, Tennessee, USA, September 1999.

[3] Colombo, O.L., Errors in Long Distance Kinematic GPS, *Proceedings of the ION GPS '91*, Albuquerque, N.M., September 1991.

[4] Colombo, O.L., and A.G. Evans, Testing Decimeter-Level, Kinematic, Differential GPS Over Great Distances at Sea and on Land, *Proceedings ION GPS '98*, Nashville, Tennessee, September 1998.

[5] Colombo, O.L., Long Range Kinematic GPS, in *"GPS for Geodesy"*, 2nd Edition, A. Kleusberg and P. Teunissen, Editors. Springer-Verlag, 1998.

[6] Colombo, O.L., Evans, A.G., Vigo-Aguiar, M.I., J.J. Benjamin, Long-Baseline (> 1000 km), Sub-Decimeter Kinematic Positioning of Buoys at Sea, with Potential Application to Deep-Sea Studies, *Proceedings ION GPS-2000*, Salt lake City, September 2000.

[7] Obana, K., H. Katao and M. Ando, Sea-floor positioning with Global Positioning System and Acoustic Link System, *The Island Arc*, Vol. 8, 245-258, 1999.

[8] Obana, K., H. Katao and M. Ando, Seafloor Positioning System with GPS and Acoustic Link for Crustal Dynamics Observation- Preliminary Result from Experiments in the Sea, *Earth, Planets, Space*, Vol. 52, 415-423, 2000.